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Eliminating a and e from these equations, we have,

$$\sin^2 \omega = \frac{2\,D - \dfrac{2\,D^2}{r} + D^2\,v^2}{r^2\,v^2}:$$

Imagine a small sphere whose center is at the point P. The motion of the comet may be directed toward any point on the surface of the half of this sphere comprised within the sphere of the sun's activity. The probability of a direction forming the angle ω with PS, the radius-vector, will be $2\pi.\sin\omega$. Hence, dividing the integral $2\pi.\int\sin\omega\,.\,d\,\omega$ by the surface of the semi-sphere we shall have the probability that the direction of the motion will be comprised within the limits zero and ω. This probability is $1 - \cos\omega$; and from the above value of $\sin^2\omega$ we easily find the expression (a).

REPETENDS.

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(*Continued from page* 15.)

Let us illustrate by an example:

Suppose $\frac{1}{17}$ is to be reduced; we know from the *second principle* that the last figure must be 7, and from (1) we get $r_2 = 12$. It is convenient to write $r_2\,q_1$, $r_2\,q_2 + m_1$, &c., in a column, thus:

$$\begin{array}{rcrcrcll}
 & & & & & & q_1 & = 7 \\
r_2\,q_1 & = & 84 & \therefore & m_1 & = & 8 \text{ and } q_2 & = 4 \\
r_2\,q_2 + m_1 & = & 56 & \therefore & m_2 & = & 5 \text{ and } q_3 & = 6 \\
r_2\,q_3 + m_2 & = & 97 & \therefore & m_3 & = & 7 \text{ and } q_4 & = 7 \\
r_2\,q_4 + m_3 & = & 91 & \therefore & m_4 & = & 9 \text{ and } q_5 & = 1 \\
r_2\,q_5 + m_4 & = & 21 & \therefore & m_5 & = & 2 \text{ and } q_6 & = 1 \\
r_2\,q_6 + m_5 & = & 14 & \therefore & m_6 & = & 1 \text{ and } q_7 & = 4 \\
r_2\,q_7 + m_{\bar{6}} & = & 49 & \therefore & m_7 & = & 4 \text{ and } q_8 & = 9 \\
r_2\,q_8 + m_7 & = & 112 & \therefore & m_8 & = & 11 \text{ and } q_9 & = 2 \\
r_2\,q_9 + m_8 & = & 35 & \therefore & m_9 & = & 3 \text{ and } q_{10} & = 5 \\
r_2\,q_{10} + m_9 & = & 63 & \therefore & m_{10} & = & 6 \text{ and } q_{11} & = 3 \\
r_2\,q_{11} + m_{10} & = & 42 & \therefore & m_{11} & = & 4 \text{ and } q_{12} & = 2 \\
r_2\,q_{12} + m_{11} & = & 28 & \therefore & m_{12} & = & 2 \text{ and } q_{13} & = 8 \\
r_2\,q_{13} + m_{12} & = & 98 & \therefore & m_{13} & = & 9 \text{ and } q_{14} & = 8 \\
r_2\,q_{14} + m_{13} & = & 105 & \therefore & m_{14} & = & 10 \text{ and } q_{15} & = 5 \\
r^2\,q_{15} + m_{14} & = & 70 & \therefore & m_{15} & = & 7 \text{ and } q_{16} & = 0
\end{array}$$

Here we stop, because the next figure will be 7, and it will commence to repeat.

We have, therefore, $\frac{1}{17} = .0588235294117647\frac{1}{17}$.

If the fraction had been $\frac{1}{13}$, we should have had

$$q_1 = 3, r_2 = 4, r_2\, q_1 = 12, r_2\, q_2 + m_1 = 9, r_2\, q_3 + m_2 = 36,$$
$$r_2\, q_4 + m_3 = 27, r^2\, q_5 + m_4 = 30 \;\therefore\; \tfrac{1}{13} = .076923\tfrac{1}{13}.$$

The operation may be very much abbreviated by taking advantage of the well known property of repetends, that after one half of the figures are obtained the second half may be found by subtracting each figure of the first half successively from 9.

We know when one-half of the digits are found because the remainder is then $d - 1$; and we know this in the above method from the fact that any number in the *column* divided by q_1 gives the corresponding remainder.

This property I demonstrate as follows:

Let $\frac{1}{d}$ be a fraction which reduces to a repetend, [verse order,
Let $q_1, q_2, q_3, \ldots q_n$ represent the successive digits of repetend in re-
Let $r_1, r_2, r_3, \ldots r_n$ " " " remainders in reverse order,
Let $t_1, t_2, t_3, \ldots t_n$ " " " terms of column,
Let $m_1, m_2, m_3 \ldots$ " " " tens " "

The column evidently stands thus:

$$t_1 = q_1$$

$$t_2 = r_2\, q_1 = 10\, m_1 + q_2 \;\therefore\; m_1 = \frac{q_1\, r_2 - q_2}{10}$$

$$t_3 = r_2\, q_2 + m_1 = 10\, m_2 + q_3$$
$$\therefore m_2 = \frac{(10\, q_2 + q_1)r_2 - 10\, q_3 - q_2}{10^2}$$

$$t_4 = r_2\, q_3 + m_2 = 10\, m_3 + q_4$$
$$\therefore m_3 = \frac{(10^2\, q_3 + 10\, q_2 + q_1)r_2 - 10^2\, q_4 - 10\, q_3 - q_2}{10^3}$$

$$t_5 = r_2\, q_4 + m_3 = 10\, m_4 + q_5 \;\therefore\; m_4$$
$$= \frac{(10^3\, q_4 + 10^2\, q_3 + 10\, q_2 + q_1)r_2 - 10^3\, q_5 - 10^2\, q_4 - 10\, q_3 - q_2}{10^4}$$

&c., &c.

Substituting in t_3 for m_1 its value from t_2, in t_4 for m_2 its value from t_3, &c., we get

$$t_1 = q_1$$

$$t_2 = r_2\, q_1$$

$$t_3 = \frac{(10\, q_2 + q_1)r_2 - q_2}{10}$$

$$t_4 = \frac{(10^2\, q_3 + 10\, q_2 + q_1)r_2 - (10\, q_3 + q_2)}{10^2}$$

$$t_5 = \frac{(10^3 q_4 + 10^2 q_3 + 10 q_2 + q_1)r_2 - (10^2\, q_4 + 10\, q_3 + q_2)}{10^3}.$$

The law is now apparent. and we can write for the nth term

$$t_n = \frac{(10^{n-2} q_{n-1} + 10^{n-3} q_{n-2} + \ldots 10 q_2 + q_1)r_2 - (10^{n-3} q_{n-1} + \ldots 10 q_3 + q_2)}{10^{n-2}}.$$

Substituting a for $10^{n-3} q_{n-1} + 10^{n-4} q_{n-2} + \ldots 10 q_3 + q_2$, we get

$$t_n = \frac{(10\, a + q_1)r_2 - a}{10^{n-2}}. \quad \text{But } r_2 = \frac{d\, q_1 + 1}{10};$$

whence we readily get

$$t_n = \frac{[(10\, a + q_1)d + 1]q_1}{10^{n-1}} \quad \ldots\ldots\ldots \quad (A)$$

Let us next find the values of the remainders:

$$r_1 = 1,$$

$$r_2 = \frac{d\, q_1 + 1}{10},$$

$$r_3 = \frac{d\, q_2 + r_2}{10} = \frac{(10\, q_2 + q)d + 1}{10^2},$$

$$r_4 = \frac{d\, q_3 + r_3}{10} = \frac{(10^2\, q_3 + 10\, q_2 + q_1)d + 1}{10^3},$$

$$\vdots$$

$$r_n = \frac{(10^{n-2} q_{n-1} + 10 n^{n-2} q_{n-2} + \ldots . 10 q_2 + q_1)d + 1}{10^{n-1}} = \frac{(10a + q_1)d + 1}{10^{n-1}}. \quad (B)$$

Dividing (A) by (B), member by member, we get

$$\frac{t_n}{r_n} = q_1 \text{ or } r_n = \frac{t_n}{q^1}. \quad \text{Q. E D.}$$

I also demonstrate the property above referred to, viz: That after one-half of the digits are found the second half may be obtained by subtracting each figure already obtained successively from 9, as follows:

Let $\frac{1}{d}$ be a fraction which when reduced to a decimal gives a repetend in which the remainder $d-1$ occurs We evidently have

$$\frac{1}{d} = .R\,\frac{d-1}{d}, \text{ and } \frac{d-1}{d} = .R'\,\frac{1}{d};$$

in which R contains the digits before reaching the remainder $d-1$, and R' the digits after reaching that remainder. But

$$\frac{1}{d} + \frac{d-1}{d} = 1;$$

hence the sum of the second members of the equation must also equal 1. R' must contain the same number of places as R, since it contains the same sequence of figures differently arranged. Now since the fractional parts when added give 1, $R + R' = 99\,.\,.\,.\,.\,.\,9$ repeated as many times as there are digits in $R + R'$, whence $R = 99\,.\,.\,.\,9 - R'$. Q. E. D.

The *second principle* may be proved as follows: As has been shown, the remainder next to the last is

$$r_2 = \frac{d\,q_1 + 1}{10}.$$

In order that this may be an integer the *units* of $d\,q_1$ must be 9; and it is evident that if the units of d be 1, then q_1 must be 9; if it be 3 then q_1 must be 3; if it be 7 then q_1 must be 7; but if it be 9 then q_1 must be 1.

In conclusion I will remark that it is improper to call repetends, as they are usually represented, *decimals*. For an example. $.\dot{3}$ is not a decimal, but should be read *three-ninths;* so $.\dot{1}\dot{6}$ should be read *sixteen ninety-ninths;* $1\dot{4}$ should be read $1\frac{4}{9}$ *tenths*, &c.

SOLUTION OF A PROBLEM.

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The following problem appeared in the *Mathematical Monthly*, (Vol. 1, p. 29), and no solution was published in that periodical:

"Show that the product of six entire consecutive numbers cannot be the square of a commensurable number."